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ELECTROMAGNETIC RADIATION

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The terms of Special Relativity are intrinsic to the model.

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Abstract
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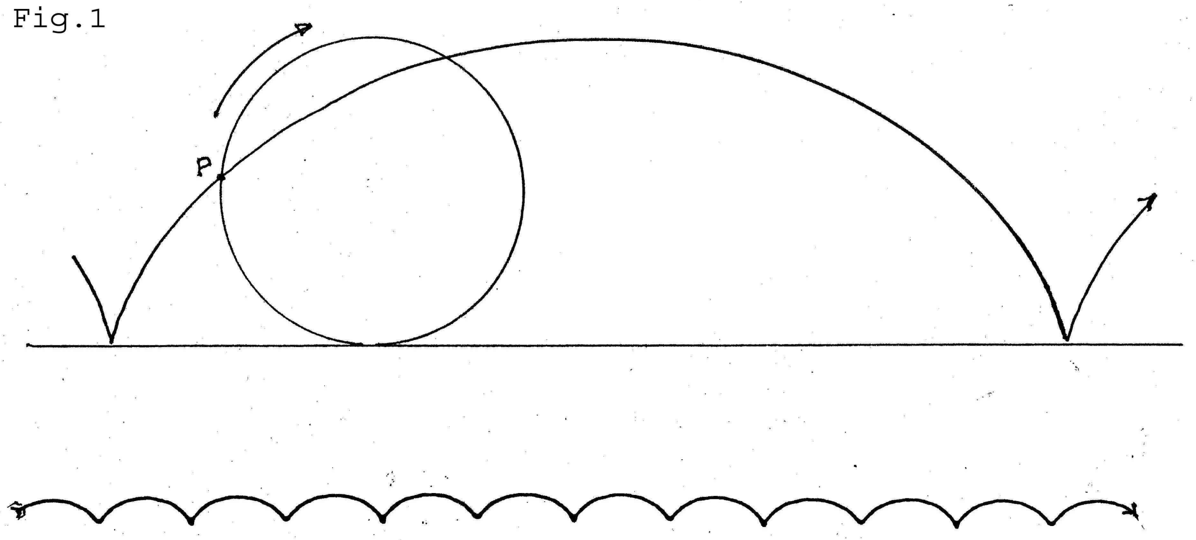
CHAPTER 1 - ORDINARY CYCLOIDAL MOTION

1 - Definitions

Ordinary cycloidal motion is defined as the motion of any point of a circumference that rolls with constant velocity on a straight line of the plane. The geometrical trajectory described by the point in the course of a single rolling is the ordinary cycloid.
It goes without saying that the motion resulting from several rollings is equipped with periodicity, and implies a translation of the point along an overall direction, which is that of the straight line on which the circumference rolls.

Fig.1



A *cycle*, both in the spatial and the temporal sense, is the single cycloidal "jump," from one extreme to the other.
More precisely, a *period* is the duration of the cycle, and the *length* of the "jump" (d) is the distance covered in each cycle between two successive extremes, which we shall call cuspidal points, or *cusps*.
Frequency (f) is the number of cycles performed in the unit of time. The greater the velocity of rotation of the circumference, i.e., of rolling, the greater is the frequency.
The *overall direction of propagation* is that of the line r on which the cuspidal points lie (or on a line parallel to it).
We also define the *instantaneous velocity* (v_i) of a generic point of the trajectory, in both the modular and the vectorial sense, represented by an arrow, tangent in that point to the trajectory, whose length indicates the modulus, and whose orientation indicates the direction and the sense.
The *useful instantaneous velocity* (v_{iu}) is the projection of v_i onto the overall direction of propagation; i.e., the component of v_i that is useful for producing advancement in that direction.
The *mean useful velocity in the direction of propagation* (v) is the mean of the useful instantaneous velocities, calculated on the basis of a whole or semi-whole number of cycles; it is equal to the "length of the jump" divided by the period ($v=d/p$).

The *tangential acceleration* (a) is the variation of the modulus of the instantaneous velocity, i.e., its derivative, represented by an arrow tangent to the trajectory, whose length gives the modulus and whose direction, coinciding point by point with that of the instantaneous velocity, qualifies it also as vector.
Initial acceleration (a_i) is the tangential acceleration at the instant in which it begins a cycle, equal, in modulus, to the tangential deceleration with which the previous cycle terminates, registered at the same cuspidal point and at the same instant.

2 - Kinematics of ordinary cycloidal motion

The instantaneous velocity varies from zero, at the beginning of the cycle, to double the mean useful velocity of propagation. Maximum velocity is attained half way through the cycle, at the peak of the cycloidal jump.
The equation for the instantaneous velocity as a function of time (with all the parameters unitary) is, in the course of each cycle:

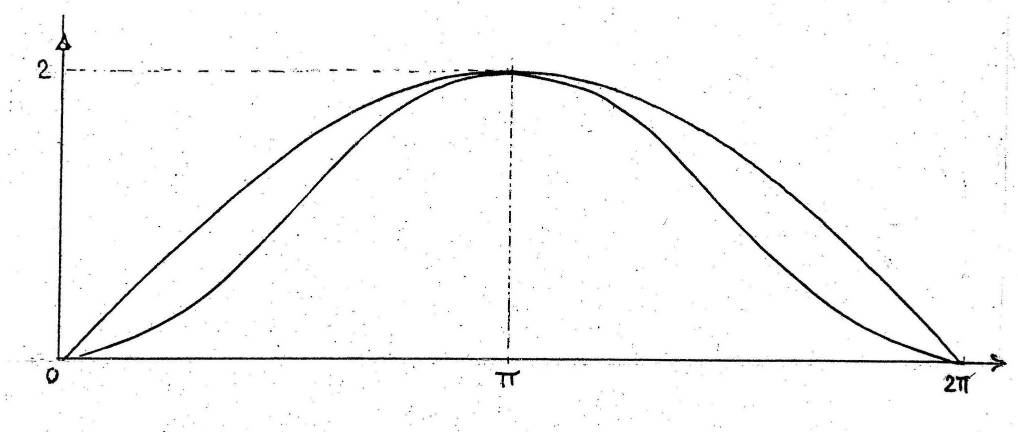
$$y=\sqrt{2-2\cos x}$$

Also the useful velocity varies between these extremes, but its values are always lower than those of the instantaneous velocity, apart from the extremes and the middle point, where they coincide.
The equation for the useful velocity, in the course of each cycle, is:

$$y=2\sin^2\frac{x}{2}$$

In Figure 2 the two graphs, that of instantaneous velocity and that of useful velocity, are referred to the same frame of axes for comparison.

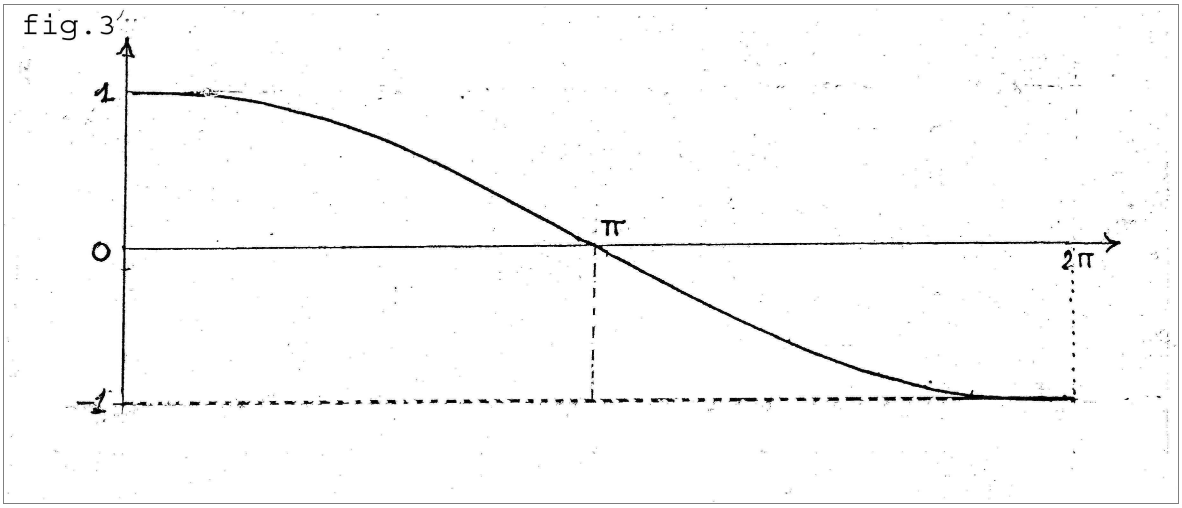
fig.2



The mean useful velocity is one half of the maximum velocity attained.
For the first half of the cycle the point accelerates, for the second half it decelerates. The maximum acceleration is the initial acceleration and the maximum deceleration is the final deceleration, relative to each of the two cuspidal points, respectively.
The acceleration is zero where the velocity is greatest.
The equation for the tangential acceleration in the course of each cycle is:

$$y=\frac{\sin x}{\sqrt{2-2\cos x}}$$

Figure 3 shows its progress.



Initial acceleration, mean useful velocity and frequency are linked by the following relation:

(1) $$a_i = 2\pi v f$$

3 - Dynamics of ordinary cycloidal motion

If, in the vacuum of elementary dynamics, the point is a material point possessing inertial mass m, and there is a vector force F applied to it capable of making it move, in what conditions is ordinary cycloidal motion obtained?
We show that:
Given a frame of reference, a material point of mass m initially at rest, to which a vector force F in rotation on a plane with constant angular velocity ω is applied, moves with ordinary cycloidal motion.

Demonstration:

m = mass
ρ = modulus of the vector force
F = force
ω = angular velocity of the rotation
a = acceleration

The dynamic hypothesis is:

$$m\,a = F(x, y, t) = (\rho \cdot \cos\omega t, \rho \cdot \sin\omega t)$$

To obtain the trajectory it is necessary to integrate this dynamic configuration twice.

$$mx'' = \rho \cdot \cos\omega t \qquad my'' = \rho \cdot \sin\omega t$$

$$x = \frac{-\frac{r}{\omega^2} \cdot \cos\omega t + \frac{\rho}{\omega^2}}{m} \qquad y = \frac{-\frac{r}{\omega^2} \cdot \sin\omega t + \frac{\rho}{\omega} \cdot t}{m}$$

If we render the parameters m, ρ and ω unitary, we obtain:

$$x = -\cos t + 1$$
$$y = -\sin t + t$$

which is the parametric equation of the ordinary cycloid of the line.

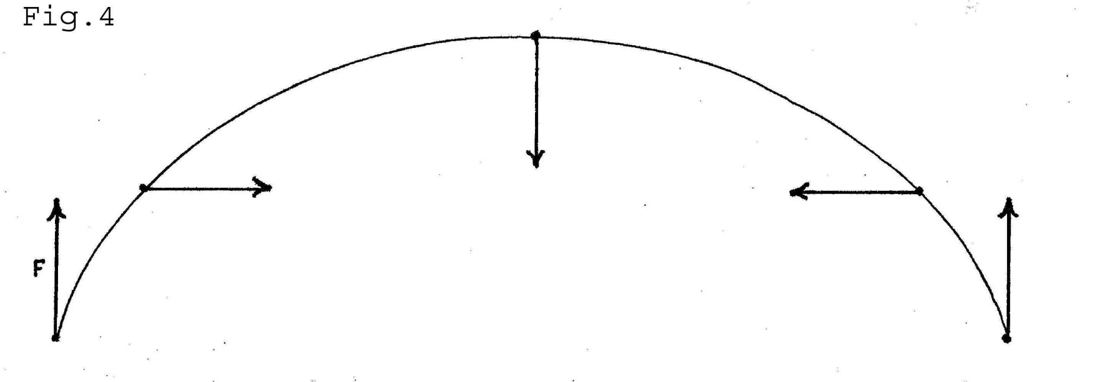
Observation

If, given a frame of reference, the initial state of rest is simpler (= has a smaller quantity of information) than the initial state of motion, and if the simplest (least "informed") way of making the direction of a vector vary over time while maintaining its modulus constant is rotation with constant angular velocity on a plane, then ordinary cycloidal motion stems from the simplest (least informed) dynamic hypothesis after the one that gives rise to uniformly accelerated rectilinear motion, which is distinguished from the former by the lack of the rotation of the force.
In other words, given a frame of reference and, in it, a material point initially at rest, the ordinary cycloidal trajectory is, dynamically speaking, the trajectory that is simplest to obtain after the rectilinear trajectory.

4 - Relations among parameters

In the course of the cycle, the arrow that represents the thrust (force F) is oriented as we see in Figure 4 in the five most significant positions, at the beginning of the cycle, at one fourth, at mid-cycle, at three fourths, and at the end.

Fig.4



We show that, given a fixed value for the mass m :

a) - Velocity of rotation (ω) - i.e., frequency (f) - being equal, increasing the thrust (F) increases the length of the jump (d) and, consequently, increases the mean useful velocity v .
b) F being equal, increasing ω , d decreases and consequently so does v .
c) Proportionally increasing F and ω , d decreases and v remains constant.

The equation that links the parameters in play is, by virtue of (1) and given $F = m \cdot a_i$:

(2) $$F = m \cdot 2\pi v f$$

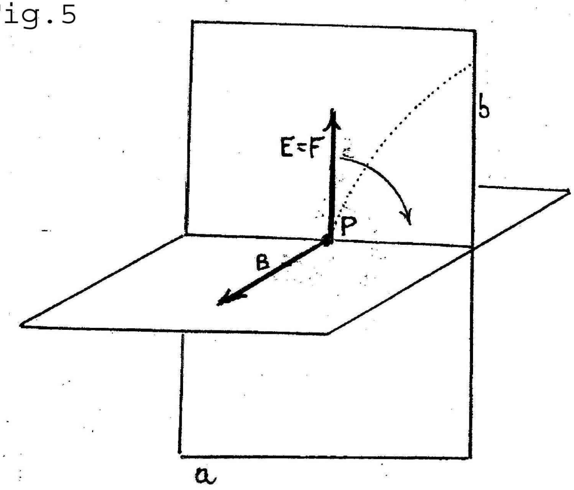
CHAPTER 2 - ELECTROMAGNETIC RADIATION

1 - The elementary constituent of electromagnetic radiation (photon)

The defining terms that will be adopted here (for the most part borrowed from the physical nomenclature in force) are purely conventional: in general the physical description of the objects and of the properties to which they refer is not given.

The elementary constituent (or elementary particle) of electromagnetic radiation (photon) is defined as a punctiform object (P in the figure) having inertia (m) to which a vector is applied (represented by the arrow F), capable of translating it in space; the vector is in rotation with constant angular velocity on a plane (ab in the figure).

Fig.5



In the figure the rotation, with its sense, is symbolized by the curved arrow; the length of the arrow is proportional to the velocity of rotation, just as the length of the rectilinear arrows is proportional to the intensity of the vectors that they symbolize. The direction and the sense of vector F are the same as those of a here not better defined vector E, *electric vector of the photon, elementary constituent of the electric field of radiation*.
The axis of rotation, i.e., the perpendicular to the plane of rotation passing through the point, is conventionally equipped with a sense. This axis is defined as the *magnetic axis*, and is represented in the dynamic diagram by another arrow, B, it too issuing from the point, orthogonal to the first arrow and to the plane of rotation, with its tip, i.e., the sense, oriented in the direction in which the rotation of the electric vector is seen to be clockwise, as in our figure.

Hypothesis on the nature and the structure of E and of F

For a moment, let us step outside our schema to hazard - parenthetically and in a first approximation - an intuitive hypothesis on the physical structure of the entities we are considering.
Think of a "perturbation" of space, localized - as far as we are concerned here - in a "small surrounding of the point", integral with the point itself and stable in the frame of the rotation. In this sense "material point" is redefined simply as *geometric center of the perturbation*. The perturbation may be described in terms of variation (as a function of its position) of the potential - always positive - of the space (where the lack of perturbation - *the vacuum* - would be described by a constant potential, with zero derivative at every point). At its central point the potential is zero (punctiform "black hole"): it has no derivative, which from all directions tends toward infinity. Such a point is thus a point of discontinuity of space, a *singularity*. The fact that the perturbation represents an elementary "electric field" possessing a direction and a sense implies that it has no radial symmetry around its center: in terms of the behavior of the derivative of the potential to tend toward

zero from all directions, this means that there is a direction and a sense (which rotate with constant angular velocity) along which the potential decreases more rapidly toward zero (and its derivative increases, in absolute value, more rapidly toward infinity).
The force that causes the center to move (dragging the surrounding structure of the potential, which is integral with it) may be seen as the effect of the unbalance due to the asymmetry of the structure itself, analogously with what occurs in fluid dynamics: a useless attempt to reinstate the perturbation's radial symmetry. The attempt is useless because, just as the carrot at the tip of the stick always maintains the same distance from the donkey in motion to reach it, so the unbalanced structure of the potential, which is rigid and integral with the center, does not modify with its motion, and thus continues to be translated.
Analogous to the terminology used for the electric vector, we attribute the oriented direction of the magnetic axis to another "localized perturbation of space" (whose nature we shall not investigate here, but which in all likelihood is due to the fact that the first perturbation is in rotation) that we call *magnetic vector of the photon, elementary constituent of the magnetic field of radiation*.

2 - The motion of the photon

By virtue of what was said in Chapter 1, in particular in Section 3, the photon (the center of the perturbation), starting off at rest in the given frame of reference, describes ordinary cycloidal motion.

We assume that:
a) - All the photons have the same inertial mass m.
b) - For each photon F and f are proportional to one another; i.e., that the relation between force and frequency (which is to say, the angular velocity of rotation of the force) is linear.
In terms of the just-formulated intuitive hypothesis, this would mean that the greater the deviation of the distribution of the potential from spherical symmetry - i.e., the greater the unbalancing of the stable perturbation - the greater is the velocity of its rotation.

It follows that - as we saw at Point (c) of Section 4 - *the mean useful velocity* (v) *is constant for the motion of all photons, independently of their frequency*. The photons with greater frequency - the ones that make more jumps in the unit of time - make their jumps proportionally shorter, so that there be no gain in the velocity (v) with which they advance as a whole. In the conditions posed there is thus an inverse relation between d (length of the jump) and f (frequency).

3 - Perceptibility of the photon's electric and magnetic fields

Let us suppose that the photon's electric and magnetic fields are "perceptible" (on the part of an "observer", an "exploratory object sensitive to the field"), and the more so the more slowly the point proceeds in the course of its trajectory. In fact, the greater the instantaneous velocity, the shorter is the time the receptor (imagined to be punctiform) of the field stays in the field itself, the less does it interact with the field and, thus, the less does it perceive it. We thus have maximum sensitivity to the fields on the part of their respective exploratory objects at the beginning (and at the end) of each "jump", when for an instant the velocity is zero.
Let us assume a mean useful velocity of displacement so great (c) that for most of the distance covered by each jump the instantaneous velocities are so high that the two fields prove imperceptible, at least where certain effects are concerned.
Let us go so far as to assume, as a working hypothesis, that the vectors are perceptible only at the cuspidal points, where velocity is annulled. However great its mean velocity, the photon will nonetheless have to present itself at its appointment with the cusps with zero velocity, which it will attain by passing through all the gradually decreasing values of velocity: the two fields will be perfectly perceptible, for an instant, only in correspondence with the cuspidal points of the cycloidal trajectory.
It follows that when both the magnetic and the electric fields are perceived they are always headed in the same respective sense, which for both will be orthogonal to the overall direction of propagation: this consideration is pertinent especially in the case of the electric vector (see Fig. 4), whose orientation varies continually in the course of a cycle.
Thus the instantaneous appearing and disappearing of a single photon will be perceived at points aligned in the direction of propagation, along a straight line, as far apart in the space between them as the length of the jump, in successive instants separated by the duration of the period of the cycloidal motion. At those points and at the instants aforesaid the double elementary perturbation E and B of the space will be localized, represented by the two vectors orthogonal to one another, and orthogonal to the direction of propagation

along which the discrete events follow one another.

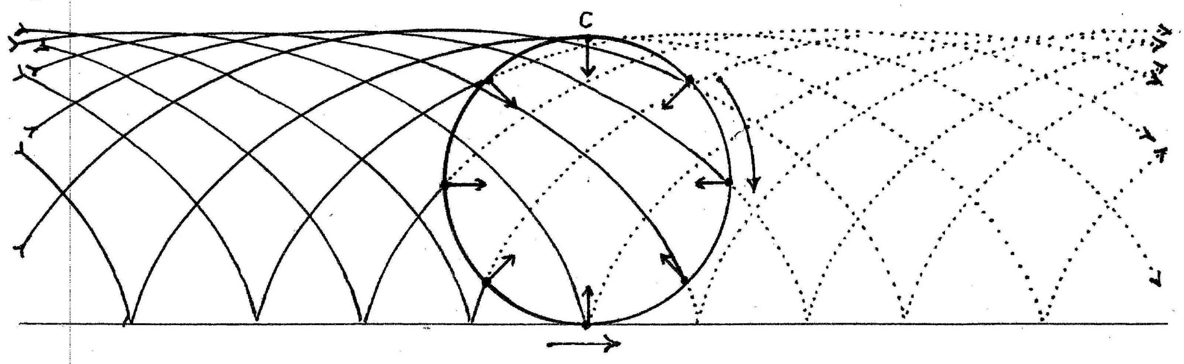
4 - Construction of electromagnetic radiation

We have seen how the single elementary constituent of radiation (the photon) moves (ordinary cycloidal motion) and the phenomenology (the succession in space and time of oriented localized perturbations E and B) to which it gives rise.
To construct a full and proper radiation, which also has wave characteristics, all we have to do is aggregate an extremely great number of the constituents ("photon gas"), making them hop in an orderly way.
To arrive at the construction of the (linearly polarized) plane wave we shall first define the progress of the perturbation along a straight line: the extension to three-dimensional space will be immediate.

4.1 - *Many points on a circumference that rolls*

On a circumference that rolls on a straight line r advancing with velocity c we fix a great number of points, not necessarily spaced the same distance apart, but in such a way that, since their distribution is also random, in the sufficiently small surroundings (arc) of any given point of the circumference they will in any case be plentiful.
Let such points be the centers of as many photons and let their electric vector be oriented toward the center of the circumference.
Consequently their magnetic vector will point up from the page in the direction of the figure's viewer who sees the circumference roll from left to right.

Fig.6



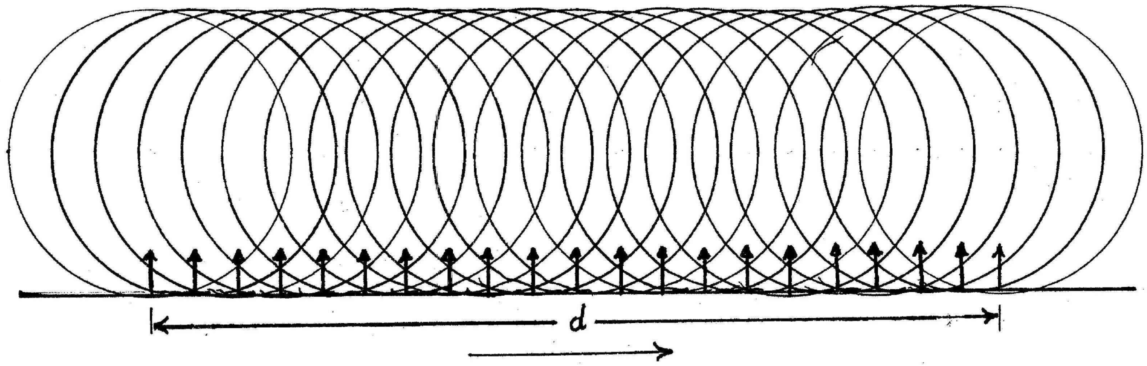
For the sake of simplicity only a few points are depicted in the figure; for each point the cycloidal trajectory has been traced (with a solid line for the trajectory already traversed and a dotted line for the trajectory remaining to be traversed).
We adopt this kinematic diagram for clarity's sake; it is evident that the identical configuration may be obtained prescinding from the circumference that rolls and exclusively employing the dynamic hypothesis of the rotating force applied to an ensemble of photons that have been emitted in an orderly way. In fact each of the positions in the figure is that which would be occupied at a certain instant by a photon emitted from a precise point at a precise instant, in flight in the course of its cycloidal motion induced by the rotating force applied to it.
For example the position (C in the figure) antipodal to the circumference's point of contact with the straight line is that of the photon that has reached its maximum velocity (2c) and is half way through its jump: we know in fact that in that position the force (vector E) is perpendicular to the trajectory and directed downward, as our diagram shows (see Figure 4).
Given what was said in Section 3 of this chapter, *our observer will see a "perturbation" translated with velocity c along the straight line on which the ideal circumference rolls*. This will not be a single perturbation that moves rectilinearly, but rather a "continual" succession along the line, in time and in space, of the "landings" of always different perturbations (in the ambit of a rolling). Each perturbation, moreover, follows not a rectilinear but rather the ordinary cycloidal trajectory which, however, is not perceived by the observer, since he can interact with the perturbation only when it is at rest relative to him.

The perturbation that advances consists, perceptively, in an "electric vector" always oriented in direction and sense orthogonal to the direction of the advance and contained in the plane of the rolling; associated with this vector is a "magnetic vector" (not shown in Figure 6, nor in the two figures that follow) with direction and sense orthogonal to the plane itself.

4.2 - *A train of circumferences that roll*

We now add a series of circumferences that roll on the same straight line, one following the other at a very close distance, such that a great number of them are in contact with the line within a sufficiently small interval fixed on it; each one, like the previous one that advanced by itself, will have a great number of "elementary perturbations" centered on it and oriented toward its center.
Let the train, measured from the point of contact of the first circumference to the point of contact of the last one (i.e., between the respective centers) be as long as the jump made by each individual photon, namely d.

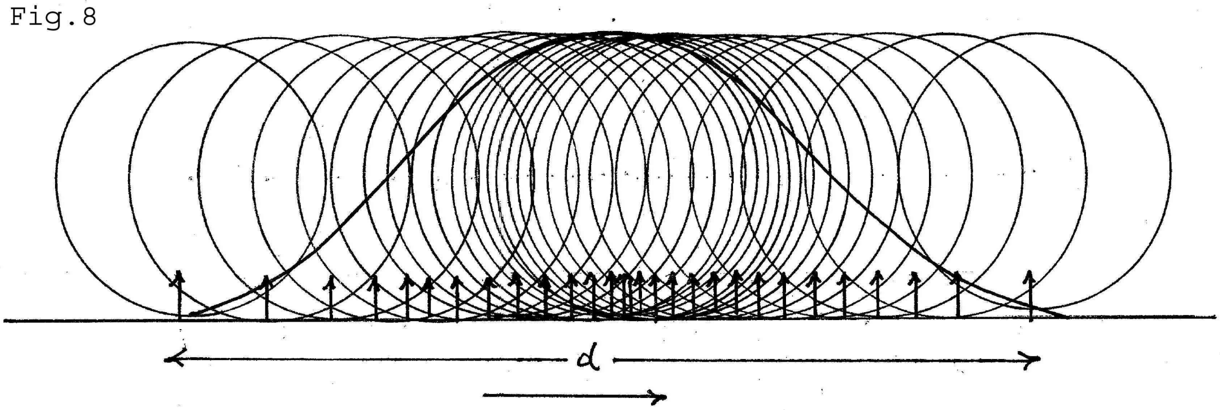
Fig.7



The point of contact of each of the great many circumferences with the line corresponds to the perturbation that is perceived by the observer from one time to the next during the rolling, with the advance of the train: *a perturbation in the form of a rectilinear segment of length d will be perceived to be translated with velocity c, constituted instant by instant by the ensemble of electric and magnetic vectors stably oriented as we stated above.*

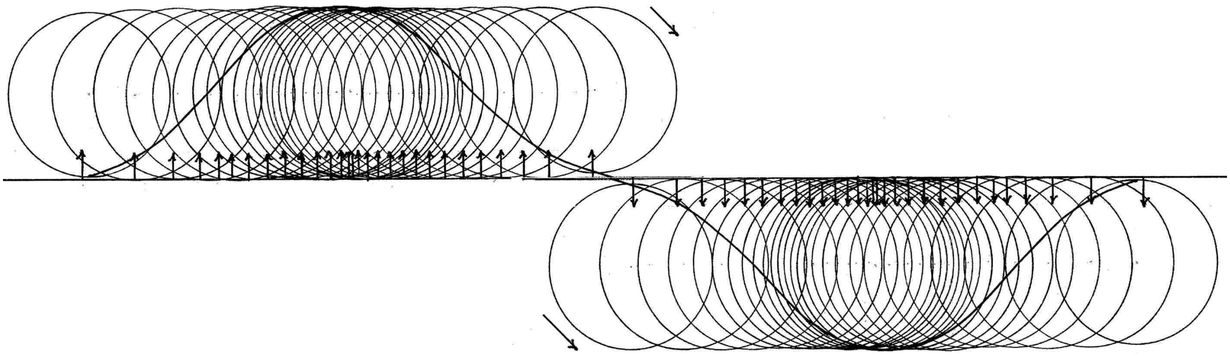
Now, let us suppose that the distribution of the circumferences that roll (the density of the points considered on each of them remaining equal) is not uniform along d, but is denser at the center and less dense toward the extremes of the segment. Let the law of the distribution be "bell-shaped", typical of normal distributions, since here in any event we have a finite domain, of the type $\sin^2 x$.
Consequently $\sin^2 x$ will be the intensity of the field perceived along the segment of length d that is translated on the direction of propagation with velocity c.

Fig.8



4.3 *Alternate trains of circumferences that roll*

Following on the train in the figure (d_1) we set another train (d_2) rolling, composed of the same number of circumferences, distributed in the same way, but this time below the line, on a segment equal in length to that of the first train.
If in the stretch d_1 the electric vector was oriented upward, in the stretch d_2 it is oriented downward; and if in the first the magnetic vector pointed up perpendicularly from the page, in the second it enters into it.



We then place, one after the other, in any quantity, other trains of length d of circumferences that roll alternately above and below the line in each train.

4.4 - *Transition to three-dimensional space: the plane wave*

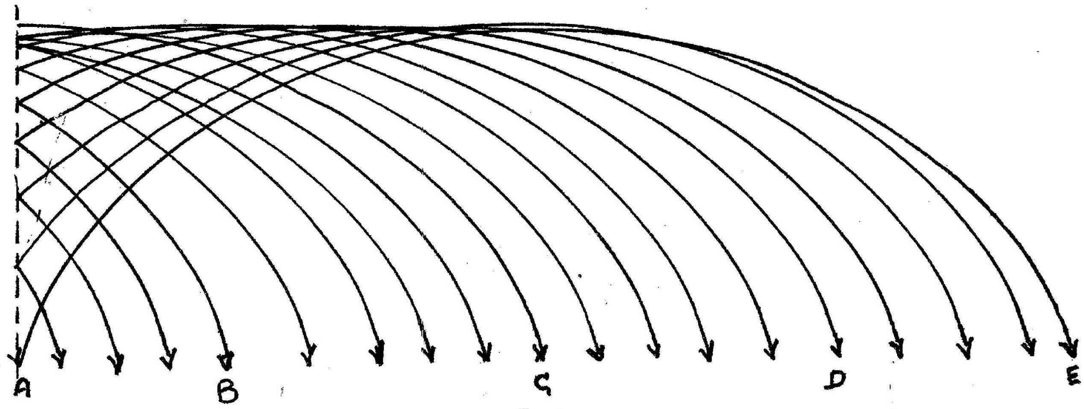
We divide the three-dimensional space in parallel sectors individuated by planes perpendicular to the direction of propagation and passing through the "nodes" (the extremities of the segments of length d), and we fill each sector - with a density distribution equal to the one fixed on the linear dimension of d - with circumferences that roll (keeping to planes parallel to that of the circumferences already given) in one sector in the same sense (clockwise) and in the next in the opposite sense (counterclockwise); which is to say that each circumference rolls, according to its sector, below or above its line, parallel to the circumference with which we began.
We have thus constructed the plane wave.
The wavelength is $2\mathrm{d}=\lambda$
The period is $2\mathrm{p}=P$
The frequency is $f/2=\nu$

5 - Emission of electromagnetic radiation

Source of electromagnetic radiation, or emission antenna, is defined as an object (not more precisely specified here) that periodically assumes two states, with a polarity reversal produced in the transition between them.
Spatial polarity implies an axis, whose orientation is constant over time.
Complete cycle, consisting in two half-phases, is defined as the occurrence of two events, after which the initial conditions for the beginning of a new cycle are reinstated. *Period* is the duration of the complete cycle, and *frequency* the number of complete cycles in the unit of time.
As a result of dynamics we shall not investigate here, the oscillation defined provokes the emission from this "antenna" of photons in ordinary cycloidal motion.
For the sake of simplicity we provisionally assume that the emission be spherically isotropic in the surrounding space, and that each photon emitted follow a trajectory of ordinary cycloids lying on a meridian plane - a plane, i.e., that contains the axis of the dipole.
Let the dimensions of the source be negligible relative to d.
Now, we assume that in the first half-phase of every cycle of the source's activity the cycloidal trajectory of the photons emitted shall have their concavity directed downward, and in the second directed upward. We also assume that the photons can leave the antenna with any velocity (understood also vectorially) included

in the interval of the instantaneous velocities assumed in the course of the ordinary cycloidal jump, so that they can then reach a value of zero velocity, at the end of the jump, at any distance from the antenna included in the interval of length d of a complete jump.

Fig.9



In Figure 9 we see the trajectories of different photons, emitted by the antenna with different velocities from a point of their cycloidal trajectory more or less distant from the conclusion of the jump.
No photon, with the first jump, can go farther than distance d; it follows that the period of the wave that will be emitted will be equal to that of the activity of the antenna.
In the course of each half-period, of duration p, the emission takes place "with continuity", from the beginning to the end of that interval of time; and, here again by dynamics we have not demonstrated, we assume that the distribution of the emission's intensity over time, probably linked to the progress of the velocity of the oscillatory motion, is the one established previously, viz., $\sin^2 x$.
For a visual rendition of this "cascade", let us analyze the emission's first half-phase in detail, breaking it down into its successive instants.

Let us consider separately the ideal cascade of photons that at the instant in which the first half-period begins detach themselves simultaneously from the antenna, each one departing from it from a different point of its cycloidal trajectory, in order to cover the entire span of positions.

This instant is represented in Figure 9. In the figure the different levels of the vertical frame from which the individual photon detaches itself is unimportant: what counts is the horizontal frame.
Strictly speaking, we should have drawn a punctiform source; but, here, we have adopted this representation in order to obtain the alignment of the landings.
Moreover, all the cycloidal paths in the figure are the "future" trajectories of the individual photons, which have been drawn to translate the information of their velocity and acceleration at the moment of detachment, when the first photons all "show themselves" outside the antenna "simultaneously". Since the figure is designed to represent a moment of time, viz., the first instant of the activity of the dipole, what is important, as we said, are the points of departure in the horizontal frame, with which a velocity and an acceleration are associated that will produce the trajectory indicated. We know that the electric field is perceived when the photon reaches the cusp of its trajectory, with zero velocity, and we have assumed that the concavities of the cycloidal path shall point downward in this first half-phase. If follows that the electric field vector that will be perceived around the antenna in the first half-phase points upward.
Let us now consider the future outcome, i.e., the actual trajectories.
The photons that show themselves outside the antenna with very low velocity in their deceleration phase (such as photon A) will immediately reach their "dead-point", making their fields felt in the vicinity of the antenna. The photons that show themselves with greater velocities, albeit they too in their deceleration phase (such as B), make their fields felt a little later and farther along, when and where they in turn reach their cuspidal points; and the greater their distance from the antenna, and thus after a longer time, the greater will be their points of departure from the conclusion of the cycle. The photons that detach themselves at maximum velocity - the velocity registered half-way through the jump (such as C) - will glide down and land at a distance equal to one half of the jump, after a time p/2; and the photons that fall even farther along, from the mid-point of this spatial interval to its extreme (such as D), will be those that detached themselves at a velocity less than maximum, but in acceleration.

The photon that closes the cascade, photon E, making the longest jump and, after a time p, coming to a halt for an instant at the distance of the length of its jump, will be the one that left the antenna with zero velocity in acceleration: this, moreover, is photon A, seen taking off instead of landing.
As the photons "touch down" they take off again immediately, no longer making their electric field be felt; and since the photons that successively touch down are gliding to a greater and greater distance, the electric field, determined moment by moment by always different photons, will "sweep" with constant velocity, always pointing upward, the space of a half-phase in the time needed by any given photon to make a complete jump.
Since we have considered only the instantaneous cascade at the beginning of the cycle, the perturbation in motion is limited to a very restricted region of space - ideally to a point, as represented in Section 4.1 of this chapter, on the contact with a straight line of a circumference that rolls - and continues to move with the same velocity and, of course, even beyond the distance of half a wavelength: once they have left the antenna, the photons continue to hop in the same way endlessly, if they meet no obstacles. When the last one has touched down, the others are in the air and will glide down farther along, one at a time and one after the other, both in the spatial and the temporal sense.

But we are just at the beginning of the half-phase emission.
After the cascade of photons emitted at the first instant (which are few indeed or, strictly speaking, none at all, given the temporal law of the emission's intensity) other (ideal) cascades will follow, for the entire duration of the half-period. Of course they are not distinct, discrete "cascades": as we said earlier, for the sake of convenience we have broken up the emission, which is to be understood as continual for the entire duration of the half-cycle.
The emission from the first instant on, in accordance with the temporal law established ($\sin^2 x$), will be on the increase for the first half of the half-phase, and on the decrease for the second half.

At the end of the half-period, at the moment in which the polarity of the antenna is reversed, the instantaneous photo of the perturbation of the space, which until then gave us a restricted region of space (a point) affected by the electric field at distance d from the antenna, now gives us an extensive perturbation, which affects the entire interval of length d beginning at the antenna, accentuated at the center and toned down toward the extremes (see the density distribution of the arrows, or the progress of the circumferences, in Figure 8, where the origin, the source, is fixed at the left extreme of the segment of length d).
To avoid misunderstandings, it is important to understand that the order in which the photons "land" one after the other, each one contributing to the construction of the half-wave that advances with velocity c while conserving its length d, effectively stems from their random emission: in the span of a half-period of the source's activity they detach themselves at all velocities (compatible with ordinary cycloidal motion: but we shall see later on how even this obligation will have to be broken), with a distribution that is in fact perfectly random over time.

In the subsequent half-period the cascade will have the opposite sense, and so forth.

6 - Generalization: the source also emits photons in non-ordinary cycloidal motion

Thus far we have taken the emission to be constituted only by photons that make ordinary jumps.
The model makes provision for a source emitting photons that make cycloidal jumps of all kinds, hence also non-ordinary (which shall presently be defined).
The generalization is right and proper if we consider the fact that ordinary cycloidal motion - the motion with cusps - represents a most particular case, dynamically speaking. To have this trajectory, it is in fact necessary that at a given instant (which will be the initial instant of the cycle) the photon have zero velocity, and there is no reason why in a physical entity this most particular condition has to enjoy special privilege.
For photons to detach themselves from a source and then glide with zero velocity to a certain distance from it, the direction of the force, at the moment of take-off, must necessarily diverge from that of the velocity in progress, given the modulus of the velocity itself, by a precise angle, and this angle must necessarily lie on the force's plane of rotation. All the possible combinations between this angle and the value of the modulus of the velocity are legible in the dynamic scheme of the ordinary cycloidal trajectory with the instantaneous velocity vector associated at every point, with its modulus and its orientation: other combinations, including those in which the velocity vector does not lie on the force's plane of rotation, belong to non-ordinary cycloidal trajectories, which differ from the ordinary trajectory because, among other things, *in a non-ordinary cycloidal trajectory at no point does the photon ever take on zero velocity.*
Thus the source has maximum freedom of emission (respecting the given restraints, viz., the sense of the

concavity for the two respective half-cycles and the bell-shaped distribution): the photons detach themselves from the source with any velocity and with any orientation relative to the force and to the plane of its rotation. We can say in advance that, with this generalization, from the standpoint of the observer at rest nothing will change relative to what we have said in the preceding sections, since he will only be able to observe the photons that make the ordinary jumps previously considered, for the instant in which they stop at the cuspidal points of their trajectory: he unconsciously selects these photons in a gas that is enormously more dense but that will not appear to him to be so, since all the other photons will never have zero velocity relative to him.

6.1 - *The dynamics of non-ordinary cycloids of the straight line in the plane*

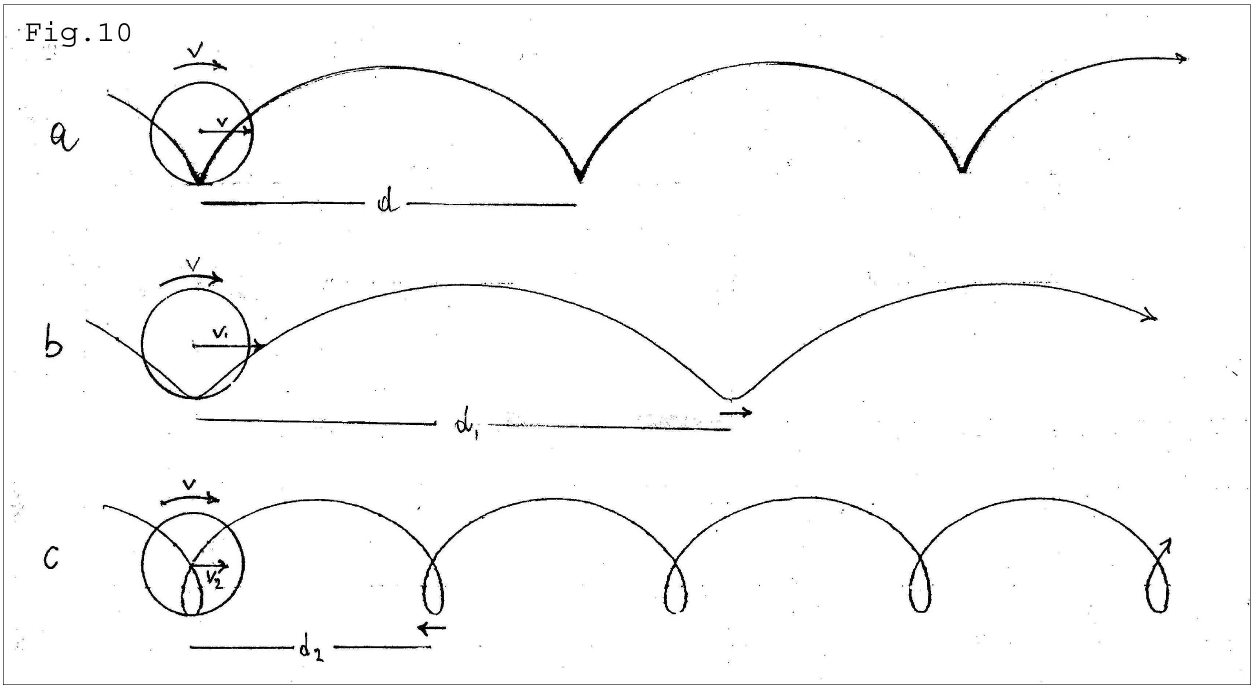
We show that if the force rotating with constant angular velocity on a plane is applied to a material point *already possessing a velocity* lying on the same plane, the trajectory described by the point is not, in general, an ordinary cycloid: it is so only at the conditions specified in Section 6. If such conditions are not satisfied (and, I repeat, we are on the plane) we shall have cycloids termed either "prolate" or "curtate", according to the direction and the sense of the initial velocity relative to those of the force vector.
Let us examine their respective kinematic constructions.

1 - Prolate (or "stretched", "lengthened") cycloids
The prolate cycloid is the trajectory of a point of a circumference that rotates on a straight line in the plane while "creeping" uniformly forward: which is to say, of a point that moves with constant velocity on a circumference that is translated with uniform rectilinear motion, with the (peripheric) velocity of the point less than that of the translation of the circumference.
Another equivalent definition, and the best known: it is the trajectory of a point within a circle that rolls (without creeping).

2 - Curtate (or "curved", "looped", "shortened") cycloids
The curtate cycloid is the trajectory of a point of a circumference that rolls on a straight line in the plane while creeping uniformly backward: which is to say, of a point that moves with constant velocity on a circumference that is translated with uniform rectilinear motion, with the (peripheral) velocity of the point greater than that of the translation of the circumference.
Another equivalent definition, and the best known: it is the trajectory of a point located on the outer extension of a ray integral with a circumference that rolls on the straight line (without creeping).

In Figure 10 their respective paths (b and c) are compared with that of the ordinary cycloid (a).

In all three cases the circumference is translated with uniform rectilinear motion toward the right while being traversed at constant velocity (v) by a point. The curved arrows indicate the (peripheral) velocity of revolution of the inner point at the center, and the rectilinear arrows applied to the centers indicate the velocity of translation of the circumferences. The lengths of the arrows are proportional to the modulus of the velocity. The curved arrows are all of the same length, to indicate that the velocity of rotation (v) is the same in all three cases. This length is equal to that of the rectilinear arrow in the case of the ordinary cycloid (a), it is greater ($v_1 > v$) in the case of the prolate cycloid (b), and less ($v_2 < v$) in the case of the curtate cycloid (c).



While the prolate cycloid is lengthened relative to the ordinary cycloid, the curtate cycloid is shortened, due to the presence of "eyelets" or "loops", in the course of which the motion is retrograde for a stretch (see the arrow drawn under one of the loops in Figure 10-c).
If d is the "length of the jump" in the case of the ordinary cycloid, $d_1 > d$ is the length of the jump in the case of the prolate, and $d_2 < d$ in the case of the curtate cycloid (lengths measured between two successive points of minimum velocity), as can be seen from the curves in the figure.
If, as in the figure,

$$v_1 - v = v - v_2$$

then

$$d_1 - d = d - d_2$$

We repeat that these non-ordinary cycloidal trajectories are distinguished from the ordinary trajectory by an important fact: at no point of such trajectories is the velocity annulled, since the "cusps" have now vanished. What is more, at the extremities of every "jump" a minimum velocity is registered, which depends, dynamically speaking, on the initial velocity inherently possessed by the point driven by the rotating force.
Getting back to dynamics, the non-ordinary cycloid is the more "shortened" or "lengthened" the greater (in one sense or in the other) is the value of the component of the initial velocity (which in the case of the ordinary cycloid is zero, and shall therefore be referred to as "excess" velocity) in the overall direction of the motion.
A notable non-ordinary cycloid is represented by the circular trajectory, the limit of the looped cycloids, an element of separation between the two classes with opposite concavity: it is a curtate cycloid that is "only loop": kinematically it is obtained, in accordance with the criterion of Figure 10, by assuming $v_2 = 0$.
The simplest way to obtain it dynamically is with an initial velocity directed orthogonally to the force vector (pointed upward) at the moment in which it is activated, oriented to the left if the rotation is clockwise, and with the modulus equal to that which one would have for the mean useful velocity of the cycloidal motion if the point were initially at rest.
As we see (in confirmation of the observation made in Section 3 of Chapter 1) it is, dynamically speaking, less simple to obtain the circular than the ordinary cycloidal trajectory.
In terms of "length of the jump" the value here is zero (d = 0), there is no overall "translation" of the motion: we can, however, continue to speak of temporal frequency and of period, which - as is the case with all the non-ordinary cycloidal trajectories - are equal to those of the ordinary cycloid, since (temporal) frequency and period depend only on the angular velocity of the rotation of the force, which up to now we have assumed, together with the force, to be equal for all the photons (monochromatic radiation).

6.2 - *Three-dimensional cycloidal trajectories*

If the "excess" velocity in course does not lie on the force's plane of rotation, we find ourselves in the presence of trajectories (which we shall term cycloidal in any case) that are far more complex: they develop in three dimensions, and do not lie on a plane as the ordinary, prolate and curtate cycloids do.
In such trajectories, too, a minimum of the velocity is periodically attained, in a direction that generally is not the overall direction of the motion, nor is it located in a plane that contains it.
For now we limit ourselves to this brief remark; we shall, however, have something more to say on the subject.

7 - Overlapping waves of all lengths and velocities

Thus the source of electromagnetic radiation emits, at random, photons that describe all the possible cycloidal trajectories, including the non-ordinary, in two and in three dimensions. Such trajectories, in their "landing" phase (we employ the term also to define the attaining of the minimum of the velocity in the course of their motion, and not necessarily zero value), order themselves in "cascades" like the above-mentioned photons that describe the ordinary trajectory.
It is understood that the photons emitted, in our hypothesis (once again: we are occupied with perfectly monochrome radiation), are "structurally" all equal, in the sense that the mass (i.e., the inertia) and the rotating force is equal for all of them, as is the velocity of rotation, which, as we have seen, is correlated with the unbalancing of the structure (which gives the force). What changes is only the initial velocity, which, being able to assume all the values, zero included, gives the cascades of the dipole the freedom to be composed of photons that make cycloidal jumps of all kinds.
To make this clear, let us fix a specific non-ordinary trajectory, for example a well-defined curtate trajectory, corresponding to a certain value of the initial velocity of the material point. The length of the jump, measured between two successive points with minimum velocity, is less than that of the ordinary cycloidal jump (but greater than that of the prolate cycloid).
In the course of a half-oscillation the dipole emits - among others - photons all of which describe the curtate trajectory, detaching themselves from the source at "all" the velocities (in the vectorial sense) that are registered in the arc of that specific "looped" trajectory, and with the same distribution $\sin^2 x$ in the time of their quantity, in such a way that the points with minimum velocity, at the extremity of the loops, are themselves also distributed in accordance with the same law in the space of a half-wavelength. In the successive half-oscillation, which completes the cycle, the cascade has its concavities turned the opposite way.
Let us consider a photon that detaches itself with the minimum of its velocity from the dipole, at the instant in which the first half-oscillation begins. In the time of a half-period of the antenna's activity this photon will "land" (will reach the successive minimum of velocity, completing its arc) at a distance from the dipole less than that at which the photon that makes the ordinary jump does, but it will land in the same time, equivalent to the common period of rotation of the force.
In the time of the period - the time employed by the photon to "land" - the first half of the complete oscillation will be exhausted, and thus when the photon will have reached its minimum velocity once again the emission of photons with the trajectory with downward concavity will have ceased, and with the successive half of the cycle the trajectory of photons with upward concavity will begin.
Hence the wavelength will be less than λ.
And it will be the less, the shorter are the individual jumps - i.e., the greater is the loop.
Here we have limited ourselves to considering, for the sake of example, a wave produced by photons that make curtate jumps of a certain length: the fact is, as we have seen, that the antenna emits photons that describe all possible trajectories (evidently not in the continuum, but with an extremely high density of values, sufficient to lend continuity to the phenomenal effects that interest us here), and thus every oscillation of the antenna produces an emission of overlapping "waves" of all possible lengths, within the limits of the field of variability of the initial velocity the photons may possess at the act of emission.
Thus "waves" of "all" lengths - λ, $< \lambda$ and $> \lambda$ - will be present, and overlapping, in the space. The wavelength λ, equal to 2d, travels with velocity c, the waves of less length with velocity less than c, and those of greater length with velocity greater than c.
The equation
Velocity = λ/P
must in fact hold, and since P is constant, the greater is λ the greater is the velocity.
We have rendered the picture far more complex but our observer has not noticed, since he sees the photon

only when it is at rest (relative to him), and this is never the case with the photons that make non-ordinary cycloidal jumps.
He perceives only the wave attributable to the photons that make the ordinary jumps. He does not interact with the others in any way. Let us say that he "detaches" from a background furrowed by an "infinity" of indistinguishable trajectories only the points at which the velocity of the ordinary cycloidal trajectories is annulled - trajectories which he can thus reconstruct theoretically; and those points (elementary constituents of the electric and magnetic fields) are distributed in the space as a wave that advances with velocity c. Contemporaneously other "waves" advance, some shorter, others longer, with velocities lesser and greater than c: we put them in quotes because if a precise value of their length is not fixed, they remain indistinguishable in the spatial "continuum" in which they are all present with their different lengths; which is to say, with the different minimums of velocity of the photons that constitute them - minimums, moreover, with different orientations.

8 - The observer in motion sees the photons with non-ordinary cycloidal motion

The observer, however, can perceive these other "waves" as well. It is sufficient that he move in the direction of the forward or of the backward movement of the points of their minimum velocity: because if he does so, at the extremes of their cycle, for an instant, the waves will have zero velocity relative to him.
Now he will no longer see the photons that make the ordinary jumps in his previous reference frame, but rather those that in that frame make the so-called non-ordinary jumps giving rise to a wave that is propagated with a velocity that differs from the c of the velocity with which he moves.

Let us recommence, for the sake of simplicity, with the individual photon.
Given a frame of reference, let the point move describing ordinary cycloidal jumps, and let the observer be at rest in the same frame: he will "see" ordinary cycloidal jumps and will measure mean useful velocity c (which he will obtain by dividing the distance d between two points that he sees successively illuminated in the space for the time p that intervenes between the two events).
If the observer moves with velocity v, we suppose first in the same sense as the overall motion of the point, in the new frame the ordinary cycloidal trajectory will be transformed into a curtate ("looped") cycloidal trajectory, with a distance between the points of minimum velocity less than d and a mean useful velocity of advance less than c: c - v, to be precise.
But our observer, who interacts only with the point at rest, will now see nothing at all.
Analogous reasoning holds for his movement (again at a velocity with modulus v) in the opposite sense: he will "see" (but he will not see them) prolate trajectories, with distance greater than d between the points with lower velocity and mean useful velocity in the direction of the propagation equal to c + v.
In the same frame of reference the point now moves describing the "looped" trajectory, advancing with a mean useful velocity $c-v$ (with, dynamically speaking, -v the suitably oriented "excess" additional velocity that the point inherently possesses), and with the distance between the extremities of two loops thus equal to $p(c-v)$.
This is what the observer at rest relative to the frame of reference would measure (if he saw the photon also when it was in motion).
If, however, he moves in the overall direction of the motion of the point, with opposite sense, at velocity v, in his new reference frame the curtate cycloid becomes ordinary: *he sees the ordinary cycloidal motion*, with mean useful velocity c ($=c-v+v$), distance between the extremities of the loops d ($=p\cdot c$), and the cusps at zero velocity.
The observer will find himself periodically, for an instant, proceeding with the same minimum velocity as the photon does when its turns backward: at that instant he sees it at rest.
By virtue of this last conclusion, if he is the observer who interacts only with things at rest - the observer who, in the new hypothesis, at rest did not see anything - he will now be able to see the dots successively illuminated, and thus measure c and d.
Exactly like when he was at rest in a frame of reference relative to which the ordinary cycloidal trajectory developed.
To give the picture fuller detail, and to broaden it, we now fill the plane with all the possible cycloidal trajectories referred to that frame, all of them directed as a whole like the two trajectories already considered, in such a way that they possess all the additional initial values possible (within the limits of the "pseudocontinuity" we find convenient).
At any velocity v (positive or negative) with which the observer moves in the given direction he will be confronted with a non-ordinary trajectory arising from that very "excess" velocity v, and, transforming it into an ordinary trajectory, he will continue to observe the same thing: a series of "dots turning on and off"

separated by spatial distance d and by temporal interval p, and thus, again, in "propagation" with velocity c. Which is equivalent to saying - if that which he sees represents everything from which he can obtain information experimentally - that he cannot say whether he is in motion or whether he is at rest, and, if he is in motion, with what velocity, direction and sense he is in motion relative to the provenance of the photon.

9 - Definitive generalization

Only the motion of the observer in the direction (x) of the overall advance of the photon has been considered here. Generalizing, we need to consider his motion in any direction whatever, not only in the domain of the non-ordinary cycloids on the plane (xy) but also in that of ordinary cycloids in three dimensions (xyz).
In that way, with the concavity of the arc always oriented in the right sense half-phase by half-phase, with the observer moving in the same direction and sense as the excess velocity, now directed in any way whatever, he will "reconstruct" the cycloidal trajectory in his frame of reference and, for an instant, he will perceive the photon at rest.

Let us now return to the wave proper.

CHAPTER 3 - SOURCE OR/AND OBSERVER IN MOTION

1 - Motion of the observer relative to the source at rest

Given a frame of reference, the observer can move in any direction and sense relative to the source at rest, just as the source can move in any direction and sense relative to the observer at rest.
We shall consider here the two most instructive cases, those of approaching and of moving away *along the fixed line joining observer and source*, with just a brief remark on movement along *any* direction.

1.1 - *Observer approaching the source along the fixed line that joins them*

The observer approaching the source perceives the wave (shorter, as we have seen) formed by photons that make the looped jumps: in his frame of reference in motion the individual jumps are perceived as ordinary, of length d, and the mean useful velocity of the advance of the photon, and thus of the wave, is, again, c: however, he measures the length λ_1 of the perturbation as that which it is in the given frame, less than λ . Precisely, if c - v is the velocity of the wave in the given frame, it will be

$$\lambda_1 = P(c-v) < \lambda$$

or, as a function of λ

$$\lambda_1 = \lambda \cdot \frac{c-v}{c}$$

Let us consider the frequency.
In the frame of reference of the source at rest the frequency (u_1) of the shortest wave, of length $P(c-v)$ that advances with the lowest velocity c - v, is the same (u) as the ordinary wave (length λ that travels at velocity c).
In fact, the frequency being equal to the velocity divided by the wavelength, we will have:

$$u_1 = \frac{c-v}{\lambda \cdot \frac{c-v}{c}} = \frac{c}{\lambda} = u$$

But in the frame of reference of the observer in motion toward the source the frequency (v_2) increases.
In fact:

$$u_2 = \frac{(c-v+v)}{\lambda \cdot \frac{c-v}{c}} = \frac{c}{\lambda \cdot \frac{c-v}{c}} = \frac{c^2}{\lambda \cdot (c-v)} > \frac{c}{\lambda} = u$$

As a function of u :

$$u_2 = u \cdot \frac{c}{c-v}$$

1.2 - *Observer moving away from the source along the fixed line*

Analogously in the case of moving away from the source: the observer will measure the length of the wave as that which objectively it is, > λ , and a frequency < ν .

$$\lambda_1 = P(c+v) > \lambda$$

or, as a function of λ

$$\lambda_1 = \lambda \frac{c+v}{c}$$

the "absolute" velocity of the wave now being $c+v$.

Let us consider the frequency.
In the frame of reference of the source at rest the frequency of the longest wave does not vary:

$$u_1 = \frac{c+v}{\lambda \cdot \frac{c+v}{c}} = \frac{c}{\lambda} = u$$

However, in the frame of the observer who moves away from the source the frequency decreases:

$$u_2 = \frac{c+v-v}{\lambda \cdot \frac{c+v}{c}} = \frac{c}{\lambda \cdot \frac{c+v}{c}} = \frac{c^2}{\lambda(c+v)} < \frac{c}{\lambda} = u$$

As a function of u :

$$u_2 = u \cdot \frac{c}{c+v}$$

1.3 - *Observer in motion in any direction*

If the observer is in motion (with velocity v) in any direction relative to the source at rest in the frame, he will be hit by the wave composed of the photons that describe, in the frame of the source, non-ordinary cycloidal trajectories resulting from an "excess" velocity (with respect to ordinary cycloidal motion) at the act of emission, of identical value v, which will not fail to appear in the population of photons emitted: this "excess" velocity will have the same direction as that of the observer, in whose frame in motion the corresponding non-ordinary cycloid (which generally develops in three dimensions) will thus become an ordinary cycloid.

We deduce, moreover, the sign and the magnitude of the Doppler effect, in accordance with the extent of the divergence of the direction of vector v from the line joining observer and source.

2 - Motion of the source relative to the observer at rest

Even if we grant that the velocity of displacement of the source must be added to the motion of each photon emitted (as occurs with the velocity of an object thrown from a vehicle in motion), so as to modify its cycloidal trajectory, further lengthening or shortening it, the result will have no impact on the population emitted: ordinary cycloidal trajectories would be transformed into non-ordinary cycloidal trajectories, but in compensation non-ordinary trajectories will be transformed into ordinary ones and so forth. Only the position of the mean value of the distribution would shift, with the form of the distribution remaining unchanged; and even granting a bell-shaped distribution that is very low and broad, as seems reasonable, the basic picture would not significantly change.

2.1 - *Source approaching the observer along the fixed line*

The observer is at rest relative to the frame of reference: let us recall that he perceives only the photons that make ordinary jumps relative to the frame.
If the source is in motion toward him, and the hypothesis advanced above holds, then the photons he perceives now are photons that described a looped trajectory when the source was at rest, and that now, with the velocity added to its motion, make normal jumps: in compensation, the photons he perceived when

the source was at rest now - again, by virtue of the added velocity - make longer jumps, and are no longer the ones he perceives now.
If our hypothesis does not hold, and the velocity of the motion of the source does not modify the cycloidal trajectories of the photons emitted, then the observer will continue to perceive the same photons - which make ordinary jumps - that he would have perceived with the source at rest.
Now, let us see what becomes of the wave.

Since the observer is at rest and sees only the photons with ordinary trajectories, it is the cascade of these photons from the source that we have to consider, keeping in mind the fact that the source is in motion (toward the observer).
The first photon that is perceptible in the course of the first half-oscillation (the "head" of the first half-wave) sets foot at the level of the source (at zero distance from it). The "head" (due, of course, to the stream of fresh photons that are landing), after a time P/2 would find itself at distance d from the source if the source were at rest, with the half-wave completely formed, of length $\lambda/2$; but since the source is moving (at velocity v) in the same direction and sense in which the wave is moving away at velocity c, when the half-wave is completely formed - again, after a time P/2 - it will turn out to be contained in a distance less than $\lambda/2$: since, in the meantime, the source has moved forward, the tail of the half-wave will be closer to the head than before, and the entire half-wave will prove to be more compact.
To have the effective length of the wave (λ_1), which will thus be less than λ , we have to subtract from λ the space covered by the source with velocity v in the time of the period P.

$$\lambda_1 = \lambda - vP = cP - vP = P\cdot(c-v) < \lambda$$

or, as a function of λ :

$$\lambda_1 = \lambda\cdot\frac{c-v}{c} < \lambda$$

Let us consider the frequency.
Relative to this wave that travels with velocity c, this time the observer is at rest and will measure a frequency:

$$u_1 = \frac{c}{\lambda_1} = \frac{c}{\lambda\cdot\frac{c-v}{c}} = \frac{c^2}{\lambda(c-v)} > u$$

i.e., as a function of u :

$$u_1 = u\cdot\frac{c}{c-v} > u$$

exactly as in 1.1.

2.2 - *Source moving away from the observer along the fixed line*

Reasoning analogously, if the source moves away from the observer - again, at velocity v - the wave will be lengthened, and we will have:

$$\lambda_1 = \lambda\cdot\frac{c+v}{c} > \lambda \qquad u_1 = u\cdot\frac{c}{c+v} < u$$

exactly as in 1.2.

Therefore the observer will be hit by a wave of a different length, less than or greater than λ , and will detect a different frequency, less than or greater than v, depending on whether the source approaches or moves

away from him.
It is exactly what occurred (also quantitatively: the shortening and the lengthening are the same) when it was he who was in motion relative to the source with velocity v, in spite of the fact that, objectively, as we have seen, the two dynamics that lead to the same result are completely different. In fact, while in the other case, in the absolute frame (that of the source at rest) the wave was, for example, short because formed by photons that made the looped jump, here, in the same frame (that in which now the source is in motion) the wave, formed by photons that make the ordinary jumps, is short because the tail has moved closer to the head due, precisely, to the motion of the source.

2.3 - *Motion of the source in any direction*

The generalization is even simpler in this case than in the previous one (Section 1.3).
The observer, who is at rest in the frame of reference, will in any case be hit by waves composed of photons that make the ordinary jumps in the frame (and thus also relative to him): we show that the length of the wave will vary according to the divergence of the velocity v (from the fixed line) with which the source is moving, remaining equal to λ when v is directed orthogonally to that line.

3 - Integral motion of source and observer along the direction of the emission

We now suppose that, relative to our frame of reference, both the source and the observer move in the same direction (that of the propagation of the wave) and sense, with the same velocity v.
This is equivalent to translating - with uniform rectilinear motion at velocity v, in the direction of the line joining source and observer - the given frame of reference relative to another frame.

3.1 - *Motion in the same sense of propagation as the radiation*

We suppose first that the sense is the same as the radiation that reaches the observer.
Since the observer moves in the same sense as the wave that reaches him, in order to be detectable the wave must be considered to be composed of photons that make the "stretched" non-ordinary jumps, and to be moving with velocity c + v in the given frame.
In this given frame, how long is the wave?
We know that if it were emitted from the source at rest it would be longer than λ , i.e.:

$$\lambda \cdot \frac{c+v}{c}$$

But it is emitted from a source in motion, in its same sense with velocity v, and thus its length is:

$$\lambda \cdot \frac{c+v-v}{c} = \lambda$$

Our reasoning is the same as in Section 2.1 (motion of the source toward the observer at rest): there, we were concerned with the wave constructed with the ordinary cycloidal trajectories, and the wave shortened starting from λ , while here we are concerned with a different, longer, wave, but the calculation is the same. Since the source moves in the same sense as the wave, the tail forms closer to the head, also here the wave shortens, but the initial length is greater and, as calculated, the length λ will be reinstated.
The observer is hit by a wave composed, in the new "absolute" frame of reference, of photons that describe "stretched" trajectories: a wave that in this absolute frame travels with velocity c + v, and is of length λ .
Since, however, he is moving with velocity v in the same direction and sense, he will measure the wave's velocity as c (and, if he could see the photons' trajectories, he would see them as ordinary).

Let us see what becomes of the frequency.
In the absolute frame (the one relative to which source and observer are translated) it will be:

$$u_1 = \frac{c+v}{\lambda} > \frac{c}{\lambda} = u$$

as a function of u :

$$u_1 = u \cdot \frac{c+v}{c} > u$$

In the frame of the observer in motion:

$$u_2 = \frac{c+v-v}{\lambda} = \frac{c}{\lambda} = u$$

3.2 - *Motion in the sense opposite that of the propagation of the radiation*

Reasoning analogously for the case in which the integral motion occurs in the sense opposite the one in which the wave is traveling, we come to the same conclusion.
In this case the wave that concerns us is composed of photons that make jumps shorter than d, describing the looped non-ordinary cycloidal trajectory: in the frame of the observer who moves in the opposite sense the trajectory becomes ordinary, since he sums his own velocity with the mean useful velocity of the photon coming toward him.
The source, however, emits a wave that is not shorter but, again in this case, is of length λ. In fact, as it emits the photons, the source moves in the sense opposite to that of the emission, and thus the "tail" of each half-phase will form farther from the head than it does in the case in which the source is at rest (a case in which, as we have seen, the wave composed of photons describing curtate trajectories proves to be shorter).
Thus the observer is hit by a wave composed, in the "absolute" frame, of photons that make the shortest jumps (with loops): a wave that moves with velocity c - v but is of length λ.
To the velocity of the wave he adds his own velocity v, and he thus measures a velocity c - v + v = c, which is the velocity of the wave in his own frame in motion.
The length will thus remain λ.
The frequency, which would be less in the absolute frame, given

$$u_1 = \frac{c-v}{\lambda} < u$$

is, in the observer's frame:

$$u_2 = \frac{c-v+v}{\lambda} = u$$

3.3 *Motion of the frame in any direction*

Utilizing the brief considerations of Sections 1.3 and 2.3 we show that in any direction and at any constant velocity in which source and observer together may be translated relative to another frame of reference (which may, in turn, be in motion relative to yet another frame, and so forth) the observer will find himself in the same conditions, from the standpoint of that which he measures, described in Sections 3.1 and 3.2:
a) He will always find the velocity of radiation to be c.
b) He will detect no Doppler effect (that is, the length of the wave remains λ, and he always finds the frequency to be ν).

4 - Conclusions

The measure of the velocity, the wavelength and the frequency of electromagnetic radiation on the part of an observer are invariant from the state of motion of the frame of reference - whose observer and source are integral (and thus at rest relative to one another) - with respect to any other frame.
If, in light of the aforementioned translation, or even in its absence, source and observer are in relative motion the one to the other, then only the relative velocity of the radiation remains unchanged: the observer becomes aware of the relative motion through the Doppler effect, but without being able to say, on the basis only of the magnitude and sign of that observation, whether it is he himself or the source that is in motion.

Appendix

I. Interference

As an electric field, the space occupied by the wave is to be seen as an "effervescence of instantaneous impulses" all directed, half-phase by half-phase, in the same direction and sense, but so dense in space and in time that they give the exploratory object sensitive to the field the perception of (both spatial and temporal) continuity. This is not unlike the way in which the density with which relatively small and relatively highly condensed discrete components present themselves gives the sensation of spatial continuity (for example of matter at different scales of observation: the scale that does not distinguish the grains of a heap of flour like the one that does not see the molecules of the air, or the one that does not resolve the stars of a galaxy).
Each impulse (due to the photon in the fleeting instant of its perceptibility) has a direction and a sense, orthogonal to the propagation of the wave, so that if "simultaneously and at the same point", "overlapping" the first impulse, an equal impulse of opposite sense is activated (another photon that will "land" coming from a cycloidal trajectory with opposite concavity), no effect will be perceived, since the two equal and contrary impulses destroy each other.
Conversely, if their senses are the same, added to one another they are strengthened, as already occurs between the photons where each one constitutes a half-wave.
Thus a region of space where the wave's electric field is perceived (analogous reasoning holds for the magnetic field, whatever it may be), if it is occupied simultaneously by an equal wave out of phase by π appears to be devoid of any perturbation. The interference, be it constructive or destructive, will be maximum with the cycloidal trajectories lying on the same plane, and minimum (zero) with the trajectories lying on planes orthogonal to one another.

II - Linear relation between the photon's energy and frequency

From the relation

$$a_i = 2\pi c f$$

which is to say, given

$$F = m a_i ,$$

$$F = 2\pi m c f$$

(where: a_i = initial tangential acceleration of the ordinary cycloidal motion, directly proportional to the force F and inversely proportional to the inertia m; c = mean useful velocity of propagation; f = frequency of the rotation of the force, which, considering the fact that each cycloidal jump corresponds to a half-wavelength, proves to be twice the frequency of the wave's complete cycle, constituted by two half-phases), if we further dimension the force as work (hence as energy) we will have the linear relation between the photon's energy and its frequency contemplated by Planck's fundamental equation.

Passing from the dynamics of the individual ordinary cycloidal jump to the wave, and given - as we said -

$$u(frequenza\ dell'\ onda) = \frac{f}{2}$$

we will have:

$$F = 4\pi mcu$$

III - Reflection of the photon

Let a here not-better-defined barrier be given, of given width l , constituting the interface between the vacuum and a here not-better-defined material body.
Let us assume that an individual photon (which makes the ordinary cycloidal jumps in the frame of the body and hence of the barrier) passes through the barrier, i.e., "is transmitted" beyond it, if it does not "land" within it (i.e., if it does not happen to find itself at zero velocity, with the cuspidal point of its trajectory): in the opposite case, it is reflected.

In the hypothesis of orthogonal incidence, it follows that:
1) - If the width of the barrier is greater than or equal to the length (d) of the jump, the photon will always be reflected.
2) - If the width of the barrier is less than the length of the jump, the photon, in accordance with the phase of its cycloidal motion with respect to the barrier, will have l/d probability of being reflected and $(d-l)/d$ probability of passing through.

In the more general hypothesis of an incidence with angle α there will be a P(α) probability of reflection, being P(O) the already-calculated probability of passing through with orthogonal incidence:

$$P(\alpha)=\frac{P(0)}{\cos\alpha}$$

IV - The transmission of the wave through matter ("transparency")

In a transparent medium the wave maintains its frequency while its velocity decreases: this is at the expense of λ , which decreases.
According to our theory, in passing through the refracting medium the photons describe non-ordinary cycloidal trajectories with loops ("curtate" trajectories).
This hypothesis justifies what is observed and explains, moreover, why the transparency - which implies non-interaction with matter - is possible.
The wave that passes through the transparent medium thus proves to be structured exactly like the one, not perceived by the observer, described in Section 1.1 of Chapter 3.

When the wave travels in the transparent medium:
1) - The velocity decreases.
2) - The wavelength decreases.
3) - The frequency does not change.
4) - The absorption decreases with the decrease of the velocity.

Since with respect to the constituents of the matter traversed (atoms etc.) the photons in non-ordinary cycloidal motion never reach zero velocity, the constituents (like the observers whom we have discussed at length) "do not see" the photons, do not interact with them (or at most to a minimum extent, which nonetheless produces absorption), neither deflecting nor blocking them.

If things are not seen in this perspective, one will be surprised to learn that the lower is the velocity of passing through a medium, the less is also the absorption (as with the diamond, with the velocity of the light that passes through it reduced to nearly a third, and absorption practically nil), since one would expect the opposite: the lower the velocity, the greater the interaction and the risk of obstruction and dispersion due to that which is met with along the way.
The fact is that the lower the photon's mean useful velocity, the larger is the loop (and, thus, the shorter is the wavelength); and the larger the loop, the higher is the minimum of velocity attained at its extremity - velocity which is directed in the sense opposite that of the propagation of the wave.
In short, the photon always moves fast (it is just that it periodically loops backward, with "dis-useful" - hence negative - velocity, and therefore advances slowly on the whole), and it does not see the atoms.
It does not interact with them and does not allow itself to be deflected or absorbed.

V - Polarization of the wave emitted

In Section 5 of Chapter 2 a provisional model of polarization was implicitly furnished: here we shall propose a more complete one.
The intensity of the emission of photons from the (ideally punctiform) source does not have spherical isotropy: if we consider any meridian plane (i.e., containing the axis of the dipole), and we call x the angle of divergence of the direction of emission on that plane with respect to the equatorial plane, let the law of the distribution of the emission be $\cos^2 x$.
Furthermore, let the cycloidal trajectories not lie in general on the meridian plane but on planes that diverge from it, according to the law $\cos^2 x$.
We assume as the direction of linear polarization that of the meridian plane, where the intensity of the emission is maximum ($\cos^2 0$), and around which the intensities corresponding to the greater angles are symmetrically distributed: intensities decreasing as far as zero value, on the perpendicular plane (plane of polarization) with a divergence of $x=\pm\pi/2$.
The nonpolarized wave is obtained by overlapping two waves polarized orthogonally to one another.
It will in fact be:

$$\cos^2 x + sen^2 x \ = \ 1$$

i.e., constant intensity of the electromagnetic field in every radial direction on the plane perpendicular to the radiation's direction of propagation.

VI - The minimum ("weak") emission: the "photon"

For our reconstruction of the wave the term "photon" refers to the elementary constituent of the radiation, the localized elementary perturbation perceptible in the infinitesimal surroundings of a single geometrical point.
The minimum emission of radiation that may be experimentally activated is emission by excitation of a single atom. Such emission, for us, nonetheless produces a series of cascades of a huge number of photons, which all describe ordinary and non-ordinary cycloidal trajectories, ordered in a limited series (given the brief duration of the atom's excitation) of successive waves (in the sense of the crest-trough modulus), radiated as described in the previous section. When these waves are collimated they can be reduced in a small "wave packet", with determinate lateral clearance and length ("coherence length"). The lateral clearance is due to the parallel band reduction technique and to the length of the duration ("de-excitation time of the atom") of the emission activity of the "monatomic antenna".
In such a case our oscillating dipole is in fact nothing other than the atom itself.
It is this minimum emission that is today called "photon": so do not be confused. When a photodetector clicks, detecting the passage of the object emitted by the single atom, this occurs because one of the photons (of ours) constituting the "packet" has been capable of setting an electron in motion - an event that has an extremely low probability of occurring. In fact, given the efficiency of the current photodetectors (about 20 percent), four times out of five the packet - albeit composed of a fantastic number of "projectiles" - does not do so.
The wave packet that strikes the two slits passes through them - both of them - in part, and the two weakest packets (which, however, conserve their original length) that are thus formed then reach the screen separately, interfering with one another.
Placing the detectors along the two paths, for the reasons we have given it will be highly improbable that they click together (what is more, the number of photons that compose the single band has been reduced): the simultaneous click has nonetheless been observed, and attributed by the theory in force to uninteresting and bothersome "noise".

Experiments such as Aspect's ("twin photons") have to be interpreted in the light of this model of the single weak emission, keeping in mind what has been said in the previous point, for experiments that make use of the state of polarization of the "photon": Bell's inequality must prove to be violated.

Translated by Giacomo Donis